Adv. appl. Clifford alg. *Online First*

DOI 10.1007/s00006-003-0000

Advances in Applied Clifford Algebras

Determination of the Biquaternion Divisors of Zero, Including the Idempotents and Nilpotents

Stephen J. Sangwine and Daniel Alfsmann

Abstract. The biquaternion (complexified quaternion) algebra contains idempotents (elements whose square remains unchanged) and nilpotents (elements whose square vanishes). It also contains divisors of zero (elements with vanishing norm). The idempotents and nilpotents are subsets of the divisors of zero. These facts have been reported in the literature, but remain obscure through not being gathered together using modern notation and terminology. Explicit formulae for finding all the idempotents, nilpotents and divisors of zero appear not to be available in the literature, and we rectify this with the present paper. Using several different representations for biquaternions, we present simple formulae for the idempotents, nilpotents and divisors of zero, and we show that the complex components of a biquaternion divisor of zero must have a sum of squares that vanishes, and that this condition is equivalent to two conditions on the inner product of the real and imaginary parts of the biquaternion, and the equality of the norms of the real and imaginary parts. We give numerical examples of nilpotents, idempotents and other divisors of zero. Finally, we conclude with a statement about the composition of the set of biquaternion divisors of zero, and its subsets, the idempotents and the nilpotents.

Mathematics Subject Classification (2000). Primary 11R52.

Keywords. Quaternion, biquaternion, idempotent, nilpotent.

Stephen J. Sangwine
School of Computer Science and Electronic Engineering,
University of Essex, Wivenhoe Park,
Colchester, CO4 3SQ, United Kingdom
e-mail: S.Sangwine@IEEE.org

Daniel Alfsmann
Digital Signal Processing Group (DISPO),
Ruhr-Universität Bochum,
44780 Bochum, Germany
e-mail: daniel.alfsmann@ruhr-uni-bochum.de

Received: January 6, 2009
Accepted: March 17, 2009