

# Space-Time Trigonometry and Formalization of the “Twin Paradox” for Uniform and Accelerated Motions

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**Abstract.** The formal structure of the early Einstein’s Special Relativity follows the axiomatic deductive method of Euclidean geometry. In this paper we show the deep-rooted relation between Euclidean and space-time geometries that are both linked to a two-dimensional number system: the complex and hyperbolic numbers, respectively.

By studying the properties of these numbers together, pseudo-Euclidean trigonometry has been formalized with an axiomatic deductive method and this allows us to give a complete quantitative formalization of the twin paradox in a familiar “Euclidean” way for uniform motions as well as for accelerated ones.

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