

Clifford Algebras and Euclid's Parametrization of Pythagorean Triples

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Abstract. We show that the space of Euclid's parameters for Pythagorean triples is endowed with a natural symplectic structure and that it emerges as a spinor space of the Clifford algebra \mathbf{R}_{21} , whose minimal version may be conceptualized as a 4-dimensional real algebra of "kwaternions." We observe that this makes Euclid's parametrization the earliest appearance of the concept of spinors. We present an analogue of the "magic correspondence" for the spinor representation of Minkowski space and show how the Hall matrices fit into the scheme. The latter obtain an interesting and perhaps unexpected geometric meaning as certain symmetries of an Apollonian gasket. An extension to more variables is proposed and explicit formulae for generating all Pythagorean quadruples, hexads, and decuples are provided.

Keywords. Pythagorean triples, Euclid's parametrization, spinors, Clifford algebra, Minkowski space, pseudo-quaternions, unimodular group, Hall matrices, Apollonian gasket, Lorentz group, Pythagorean quadruples, n -tuples.

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