

# An Introduction to Commutative Quaternions

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**Abstract.** - A Scheffers theorem states that for commutative hypercomplex numbers the differential calculus does exist and the functions can be introduced in the same way as they are for a complex variable. This property could open new applications of commutative quaternions in comparison with non-commutative Hamilton quaternions.

In this article we introduce some quaternion systems, their algebraic properties and the differential conditions (Generalized Cauchy-Riemann conditions) that their functions must satisfy.

Then we show that functional mapping, studied in geometry associated with the numbers, does have the same properties of conformal mapping performed by functions of a complex variable. We also summarize the expressions of the elementary functions.

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Received: April 15, 2005

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The authors wish to thank the friend and colleague Mario Lizzi for his helpful suggestions and fruitful discussions.

Accepted: July 29, 2005