

# Biquaternion (Complexified Quaternion) Roots of -1

Stephen J. Sangwine

**Abstract.** The roots of -1 in the set of biquaternions (quaternions with complex components, or complex numbers with quaternion real and imaginary parts) are derived. There are trivial solutions (the complex operator, and any unit pure real quaternion), and non-trivial solutions consisting of complex numbers with perpendicular pure quaternion real and imaginary parts. The moduli of the two perpendicular pure quaternions are expressible by a single parameter by using a hyperbolic trigonometric identity.

Stephen J. Sangwine  
Department of Electronic Systems Engineering,  
University of Essex, Wivenhoe Park,  
Colchester, CO7 9EU, United Kingdom.  
e-mail: [S.Sangwine@IEEE.org](mailto:S.Sangwine@IEEE.org)

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