Gravitational field energy approach to interaction from START

Jaime Keller *

Departamento de Física y Química Teórica,
Facultad de Química Universidad Nacional Autónoma de México
CP 70-528, 04510, México D.F., MEXICO
(permanent address) and
Center for Computational Materials Science
Technical University of Vienna
Gumpendorferstrasse 1A, A-1060, Vienna, Austria

Received 3 July 2006, accepted 13 July 2006

Key words 5-D quadratic space, gravitational field energy, START, Theory of Gravitation.
Subject classification 15A63, 15A66, 11E88

Gravitation can be casted as a mathematical structure and a physical theory by itself. The steps are: to define the interacting Fields (bodies), to define the interaction and its properties, to define the energies and forces involved, to show the self consistency of the mathematical structure involved, and a discussion of the (proportionality) relation between gravitational and inertial mass. The mathematical frame for this development is a 5-D quadratic space reference manifold: space, time and action (START). This requires the introduction of Principles (in the theory analytical mathematical quantities defined by their use) and the derivation of Laws (mathematical relations among the derivatives of of the basic fields). All known properties of the physical manifestation of gravitation are self-consistently derived.

1 Gravitation

Gravitation is an example of a physical theory where the description of nature is based both on direct observation of phenomena as seen by the observer and a collection of theoretical variables which can be either semi-intuitive (as the concept of force) or totally theoretical (as the concept of work) or primitive as space, time, material objects, system, where the use defines the concept itself.

The primitive concepts are defined in a tautological cycle: space is related to the loci of material objects, material objects are defined as that which occupies a volume in space, and time is perceived as a parametrization of the succession of changes for the material objects and space. For a mathematical theory of gravitation the best point of departure is to define a manifold describing these quantities.

As mentioned in the abstract our presentation will cast Gravitation as a physical theory by itself: define the basic manifold, define the interacting bodies, define the interaction and its properties, to define the energies and forces involved, and finally show the self-consistency of the mathematical structures involved.

2 Gravitation from START

The acceptance of space and time as primitive concepts, parametrized by a 4-D manifold (x, y, z, t say), is now universal in physics. Here matter and radiation are presented mathematically as a density of energy in space.

For a static matter–radiation distribution the space–time density of action is trivially equivalent to a density of energy $\varepsilon/\Delta x \Delta y \Delta z$ in space for a given observerThe next step is to define a space–time–action 5-D manifold. This is a generalization of the quadratic form which historically started with the Pythagorean formulation. Schematically:

* keller@cms.tuwien.ac.at

.Quadratic form of differential interval	Dimension/Op.	Basic Group
$(dl)^2 = (dx)^2 + (dy)^2 + (dz)^2 \qquad (\triangle t)$	3-D ∇, ∇^2	Galileo
$(ds)^2 = (cdt)^2 - ((dx)^2 + (dy)^2 + (dz)^2)$	4-D $D, \Box^2$	Poincaré
$(dS)^2 = (cdt)^2 - \left(dx^2 + dy^2 + dz^2\right) - dw^2$	5-D K, K^2	START

where we are defining for the action and for its differential

$w = \kappa_{(0)} a$, $\kappa_{(0)} = d_{(0)}/h = c/E_{(0)}$	$dw^2 = (dw)^2 = \sum_\mu \left(dx_\mu \left(\partial a/dx_\mu\right)\right)^2$

here l, (x, y, z), c, h, w, a, $E_{(0)}$ are distance, distance components, vacuum speed of light, Planck's constant, distance equivalent to action, action of a system and characteristic energy of the system respectively. Notice that in a physical description of motion $(dx_\mu)^2 = (dt\,(\partial x_\mu/\partial t))^2 = (dtv_\mu)^2$ using the velocities v_μ as fundamental quantities and time as the basic parameter (here $(\partial a/dx_\mu) = p_\mu$ energy-momentum defining ACTION and considering space-time as a fundamental frame of reference).

There is a nesting from the general *space-time-action* manifold $\{$with quadratic form Q_{Form} which induces a Clifford Algebra $Cl_{1,4}\}$.

Once the basic geometry is defined Physics is introduced through the: **START Relativity Principle** and the: **START Description Principle**

• "**All trajectories describing physical objects are null for all observers**" • "**The vacuum Speed of Light is c for all observers**" • "**Physical objects are described by bundles of null trajectories**" •"**The environment of physical objects is described by interaction potentials and chemical potentials**"

The null trajectories condition reduces the five dimensions to effective four dimensions. The interaction potentials describe energy-momentum being shared.

Our geometrical structure allows the development of a new, deductive scheme, formalism. For this purpose we need a minimal set of FUNDAMENTAL PRINCIPLES and POSTULATES, in such a form to obtain a comprehensive theory for Physics:

START Relativity (the laws of physics are invariant under a 5-D Poincaré group which includes 5-D Lorentz transformations);

Existence (physical objects are represented by energy densities);

Least Action (physically acceptable 'trajectories' correspond to null, optimal possible, trajectories in START);

Quantized Exchange of Action (we can define systems or subsystems as those among a quanta of action can be exchanged) and,

Choice of Descriptions (we should allow all useful physical models to be employed and properly based in START).

2.1 Action Density and its partial derivatives: energy–momentum

Multivector Representation. The base space $\mathbb{R}^5$ corresponds to the real variables set $\{ct, x, y, z, \kappa_0\alpha\} \leftrightarrow \{x^U; U = 0, 1, 2, 3, 4\}$ that is: time, 3-D space and action (all in units of distance, introducing for time the universal speed of light in vacuum c and, for action, the system under observation dependent $\kappa_0 = \lambda_{Compton}^{\text{system with energy } m_0c^2}/h = 1/m_0c$ where $\lambda_{Compton}^{\text{system with energy } E} = hc/E$ and $E_0 = m_0c^2$). In the most common approach to physics time is usually taken as an independent evolution coordinate and action (from matter and interaction) is distributed in space, then we need to consider the functions $x(t), y(t), z(t)$ and $w(t, x, y, z) = \kappa_0\alpha(t, x, y, z)$. The linear forms for the description are the nested vectors

$$\begin{array}{ll} dS = \sum_U dx^U e_U \,;\, U = 0,1,2,3,4 & 5-D \\ ds = \sum_\mu dx^\mu \mathsf{e}_\mu \,;\, \mu = 0,1,2,3;\ \mathsf{e}_\mu = e_4 e_\mu & 4-D \\ d\mathbf{x} = \sum_i dx^i \mathbf{e}_i \,;\, i = 1,2,3 \,;\, \mathbf{e}_i = \mathsf{e}_0\mathsf{e}_i & 3-D \end{array}$$

where the nesting obeys $e_4 dS = \sum_U dx^U e_4 e_U = ds - d(\kappa_0 \alpha)$ and $\mathrm{e}_0 ds = \sum_\mu dx^\mu \mathrm{e}_0 \mathrm{e}_\mu = d(ct) + d\mathbf{x}$, members of a Clifford algebra generated by the definition of a quadratic form

$$\begin{aligned} dS^2 \equiv (dS)^2 &= \left(\sum_U dx^U e_U\right)^2 = \sum_{UV} g_{UV}^{START} dx^U dx^V \ , \\ g_{UV}^{START} &= diag\,(1,-1,-1,-1,-1) \ , \ e_U e_V = -e_V e_U \\ e &= e_0 e_1 e_2 e_3 e_4 = -e^\dagger \qquad e_U e = e e_U \text{ with } e^2 = 1 \end{aligned}$$

The quadratic form which is more relevant for Physics considers that observable objects are extended in space and then an action density field α in space-time is required.

We also have the corresponding 5-D vector derivation $K = \sum_U e_U \partial^U$ and its scalar square $K^2 = \sum_U \partial_U \partial^U$. And the (in START) space-time Laplacian operator $\Box = \sum_\mu e_\mu \partial^\mu$ such that along $\mathbf{b} = \sum_\mu b^\mu e_\mu$ the directional change operator is $d\mathbf{b} \cdot \Box = \sum_\mu [db^\mu \partial^\mu]$ (we apply four times for $\mathbf{b} = cte_0, xe_1, ye_2, ze_3$). Then we can obtain the sum of the four mutually orthogonal **directed** changes of the action w:

$$dw = \sum_\mu [(\partial^\mu w(\mathbf{x},t))\, dx^\mu]\, e_\mu = \kappa_0 \sum_\mu [(p_\mu)\, dx^\mu]\, e_\mu$$

the contributions $(\Sigma_\mu ([(p_\mu)\, dx^\mu]\, e_\mu))^2$ to the action differential square allow to write for the START squared differential ($\kappa_0 p_\mu$ is adimensional and $e_\mu^2 = g_{\mu\mu}^{(0)}$)

$$\begin{aligned} (dS)^2 &= \left(1 - (\kappa_0 p_0)^2\right)(cdt)^2 + \\ &- \left(\left(1 - (\kappa_0 p_1)^2\right)(dx)^2 + \left(1 - (\kappa_0 p_2)^2\right)(dy)^2 + \left(1 - (\kappa_0 p_3)^2\right)(dz)^2\right), \end{aligned}$$

for variable p_μ we can speak in terms that **"energy-momentum** $p_\mu(\mathbf{x},t)$ **'generates' curvature in a 4-D space-time subspace of** $(dS)^2$ **"**. In general

$$\begin{aligned} dS^2 &= g_{UV} dx^U dx^V \to g_{\mu\lambda}(\mathbf{x}) dx^\mu dx^\lambda \\ g_{\mu\lambda}(\mathbf{x}) &= g_{\mu\lambda}^{(0)} - \vartheta_{\mu\lambda}(\mathbf{x})\,;\, U,V = 0,...,4\,;\, \mu,\lambda = 0,...,3 \end{aligned} \tag{1}$$

presenting $(dS)^2$ as a (locally embedded in 5-D) 4-D curved space.

A system to be studied is characterized by an action distribution, a scalar density at each point of space–time. The total density of action is a sum of contributions, which for description purposes, can be called the **constitutional action** corresponding to the carriers mass and the **interaction part of the action**, corresponding to a description dependent part.

This paper analyzes gravity, then it centers on the system-environment interaction part. Within the theory it is fundamental to include the vector e_4, $(e_4)^2 = -1$ related to the action density and then to the energy and momentum densities.

2.2 A Covariant Extension of Newtonian Gravitation

Let us formally show that the gravitation (covariant in space-time) equations and their standard textbook form are analytical properties of the third derivatives of the action density attributed to a test carrier as induced by a collection of interacting carriers (carriers for which the action and the energy–momentum is shared but partitioned).

Then if in the reference frame of a given observer the induced action density, denoted by $a_m(X)$, per unit "charge" of a test carrier (⇒puc , in this case the charge responsible for the interaction is the mass), at space–time point $X = x^\mu e_\mu$ (here $\mu = 0,1,2,3$ and $x^0 = ct$ and the space vectors $\mathbf{q} = q^i \mathrm{e}_i = q_i \mathrm{e}^i$, $i = 1,2,3$ are written in bold face letters), then the related energy density (puc) and the induced momentum density, **per unit charge of the test carrier**, would be

$$\mathfrak{E}_e(X) = \frac{\partial a_e(X)}{\partial t}, \qquad \mathbf{p}_e = p_{e,i}\mathrm{e}^i = \left(\frac{\partial a_e(X)}{\partial x^i} + \Delta_R\, p_{e,i}\right)\mathrm{e}^i, \tag{2}$$

and the, by definition, gravitational field intensity $\mathbf{E}$ is the force (puc) corresponding to this terms

$$\mathbf{E} = (\frac{\partial \mathfrak{E}_e(X)}{\partial x^i} + \frac{\partial p_{e,i}}{\partial t})\mathrm{e}^i = \nabla \mathfrak{E}_e(X) + \frac{\partial \mathbf{p}_e}{\partial t},$$

with time dependence

$$\frac{\partial \mathbf{E}}{\partial t} = (\frac{\partial^2 \mathfrak{E}_e(X)}{\partial t \partial x^i} + \frac{\partial^2 p_{e,i}}{\partial t \partial t})\mathrm{e}^i = \frac{\partial^3 a_e(X)}{\partial t \partial x^i \partial t}\mathrm{e}^i + \frac{\partial^2 \left(\Delta_R p_{e,i}\right)}{\left(\partial t\right)^2}\mathrm{e}^i.$$

Otherwise, by definition of interacting carriers, we have added in (2) the term $\Delta_R\, p_{e,i} e^i$ as the effect of the conservation of **interaction transverse momenta** between the field representing the rest of the carriers with that sort of charges. In START this is, by definition, the origin of a geomagnetic field intensity $\mathbf{B} = B_k e^k$ that will appear as the curl of the momentum (puc) of an interaction field acting on a carrier, the space axial vector

$$\mathbf{B} = \left(\frac{\partial p_{e,i}(X)}{\partial x^j}\right)\mathrm{e}^j \times \mathrm{e}^i = \nabla \times \mathbf{p}_e,$$

with time dependence

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{\partial^2 p_i(X)}{\partial t \partial x^j}\mathrm{e}^j \times \mathrm{e}^i.$$

The space variation of $\mathbf{E}$, including for this carriers the **interaction transverse momenta**,

$$\nabla\, \mathbf{E} = \nabla \cdot \mathbf{E} + \nabla \times \mathbf{E}, \tag{3}$$

will then also include a transversal (rotational) term

$$\nabla \times \mathbf{E} = \frac{\partial^2 p_j(X)}{\partial x^i \partial t}\mathrm{e}^i \times \mathrm{e}^j = -\frac{\partial \mathbf{B}}{\partial t}. \qquad \text{(2nd Grav Equation)}$$

Relation which is the direct derivation in START of the gravitational equivalent of a well known Maxwell equation. The scalar term $\nabla \cdot \mathbf{E}$ being a divergency of a vector field should be defined to be proportional to a source density

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_g}\rho = \sum_i \left(\frac{\partial^3 a_e(X)}{\partial x^i \partial x^i \partial t}\right) = \frac{\partial}{\partial t}\nabla^2 a_e(X), \qquad \text{(1st Grav Equation)}$$

and will be given full physical meaning below.

For the space variation of $\mathbf{B}$ we have $\nabla\, \mathbf{B} = \nabla \cdot \mathbf{B} + \nabla \times \mathbf{B}$. The first term vanishes identically in our theory because it corresponds to the divergence of the curl of a vector field

$$\nabla \cdot \mathbf{B} = 0, \qquad \text{(3rd Grav Equation)}$$

while the last term, using $U \times V \times W = V(U \cdot W) - (U \cdot V)W$

$$\nabla \times \mathbf{B} = \nabla\left(\nabla^2 a_e(X)\right) - \nabla^2 \mathbf{p}_e = \mu_g(\mathbf{J} + \epsilon_g \frac{\partial \mathbf{E}}{\partial t}), \qquad \text{(4th Grav Equation)}$$

where the additional dimensional constant μ_g is needed to transform from time units (used in the definition of a current $\mathbf{J} = \nabla\left(\nabla^2 a_e(X)\right)/\mu_g$) into distance units and $\epsilon_g \mu_g$ will have then units of T^2/D^2 or inverse velocity squared, in fact (see below) c^{-2}. The 4th Grav Equation, defining $\mathbf{J}$, is related to a Lorentz transformation of the 1st Grav Equation. There is also, see below, an energy-density $U = -(\epsilon_{g0}/2)\mathbf{E}_g^2$ and an energy-flux-density for the field $\mathbf{R} = c[\mathbf{E}_g \times \mathbf{H}_g]$.

A comprehensive theory of gravitation should use this formal structure, by construction is a one-to-one map of the electromagnetic Maxwell equations and effects. This is a universal interaction structure, for each type d of charge there is a pair (ϵ_d, μ_d) of coupling constants.

The derived gravitational interaction equations extend and are formally equivalent to the original Newton equations, then they are: first local equations and second linear in the sources (ρ and $\mathbf{J}$).

Both the 4th Grav Equation, defining $\mathbf{J}$, related to a Lorentz transformation of the 1st Grav Equation, defining ρ,can immediately be integrated. The space divergence of a non-solenoidal vector field like $\mathbf{E}$ is immediately interpreted as its 'source' given that in the volume element $S\Delta\mathbf{x}$

$$\Delta\mathbf{E} = (\nabla\cdot\mathbf{E})\,S\Delta\mathbf{x},$$

and this equation is integrated using the standard geometric theorem that the volume integral of a divergency $\nabla\cdot\mathbf{E}$ equals the surface integral of the normal (to the surface) component of the corresponding vector field $\mathbf{n}\cdot\mathbf{E}$. Where $\mathbf{n}$ is a unit vector perpendicular to the surface S (in the text-book formula below $S = 4\pi r^2$ corresponding to an integration sphere of radius r containing a spherically symmetric source density $\rho(r)$ generating a force field per unit charge $\mathbf{E} = E(r)\frac{\mathbf{r}}{r}$) of the integration volume $V = 4\pi r^3/3$:

$$\int_V (\nabla\cdot\mathbf{E})\,dV = \int_V \frac{4\pi}{\epsilon_g} m(r')r'^2 dr' = \frac{1}{\epsilon_0}M = \int_S E(r)\frac{(\mathbf{r}\cdot\mathbf{n})}{r}dS = 4\pi r^2 E(r)$$
$$\mathbf{E} = E(r)\frac{\mathbf{r}}{r} = \frac{M}{4\pi\epsilon_g r^2}\frac{\mathbf{r}}{r} = \frac{GM}{r^2}\frac{\mathbf{r}}{r}, \qquad \mathbf{r}\cdot\mathbf{n} = r.$$

That is: *the inverse square law of the Newtonian (and Coulombic) forces are geometrical consequences of the definition of interaction among "charged" carriers*. But this is not a derivation of the values of the *Newtonian and Coulombic* constants G and ϵ_0.

2.3 The Energy of the Gravitational Field

By construction the static gravitational field is conservative, and written in terms of a scalar potential: $\mathbf{E} = -\nabla\phi$. The gravitational force on a mass m is written $\mathbf{f} = m\mathbf{E}$. The work *the observer* would have to do against gravitational forces in order to move the mass from point P to point P' is $W = -\int_P^{P'}\mathbf{f}\cdot d\mathbf{l}$. No work is needed to place the first mass in position at $\mathbf{r}_1$. In order to bring the second mass into position at $\mathbf{r}_2$, we have to do work $W_2 = -G\frac{m_2 m_1}{|\mathbf{r}_2-\mathbf{r}_1|}$ against the gravitational field generated by the first mass. This result can easily be generalized to N masses: $W = -G\sum\limits_{i=1}^{N}\sum_{j>i}^{N}\frac{m_i m_j}{|\mathbf{r}_i-\mathbf{r}_j|}$. Consider the potential energy of a continuous mass distribution. Write

$$W = \frac{1}{2}\int\rho\phi d^3\mathbf{r},\ \text{and}\ \phi(\mathbf{r}) = -\frac{1}{4\pi\epsilon_{g0}}\int\frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3\mathbf{r}',$$

where $\phi(\mathbf{r})$ is the scalar potential generated by a continuous mass distribution. Substitute $\rho = \epsilon_{g0}\nabla\cdot\mathbf{E}$ to write

$$W = \frac{\epsilon_{g0}}{2}\int\phi\nabla\cdot\mathbf{E}d^3\mathbf{r}.$$
$$\nabla\cdot(\mathbf{E}\phi) = \phi\nabla\cdot\mathbf{E} + \mathbf{E}\cdot\nabla\phi.\ \text{and}\ \nabla\phi = -\mathbf{E},$$

to obtain

$$W = -\frac{\epsilon_{g0}}{2}\left[\int\nabla\cdot(\mathbf{E}\phi)\,d^3\mathbf{r} - \int\mathbf{E}^2 d^3\mathbf{r}\right] = -\frac{\epsilon_{g0}}{2}\left(\int_S\phi\mathbf{E}\cdot d\mathbf{S} + \int_V\mathbf{E}^2 dV\right),$$

here V is a large volume which encloses all of the masses, and $\mathbf{S}$ is its bounding surface where $\phi\mathbf{E}$ decreases as $1/r^3$. Then W reduces to

$$W = -\frac{\epsilon_{g0}}{2}\int\mathbf{E}_g^2 d^3\mathbf{r},$$

where the integral is over all space. The (negative) potential energy of a continuous mass distribution is stored in the gravitational field. By construction we then find that a gravitational field possesses an *energy density*

$$U = -\frac{\epsilon_{g0}}{2}\mathbf{E}_g^2.$$

The inertia of the energy U is also a consequence in our theory.

2.4 The Action of a Body in a Gravitational Field

For the massive carrier field at rest $\mathcal{E}^2 = (\mathcal{E}_0 + \Delta\mathcal{E})^2$, where $\Delta\mathcal{E}$ is a gauge-free energy contribution and $\mathcal{E}_0 = m_0 c^2$.

The concept of test carrier in gravitation, even in general relativity for example in the Schwarzschild solution, is compatible with the Newtonian limit for the interaction gravitational energy

$$\Delta\mathcal{E}\left(r\right) = -m_0 \frac{GM}{r},$$

where M is the mass of 'the source' we are exploring with the test particle. Consider the changes in energy-momentum arising from the interaction:

$$\begin{aligned}\mathcal{E}^2 - \mathcal{E}_0^2 &= \mathcal{E}_0^2 + 2\mathcal{E}_0\Delta\mathcal{E} + (\Delta\mathcal{E})^2 - \mathcal{E}_0^2 = (pc)^2 - (p_0 c)^2 \\ &= 2\mathcal{E}_0\Delta\mathcal{E} + (\Delta\mathcal{E})^2 \rightarrow -2m_0 c^2 m_0 \frac{GM}{r} + \left(m_0 \frac{GM}{r}\right)^2, \\ p^2 = (m_0 v + \triangle p)^2 &= \left((m_0 v)^2 - 2m_0^2 \frac{v}{c}\frac{GM}{cr} + \left(m_0 \frac{GM}{cr}\right)^2\right)\end{aligned}$$

and space spherical coordinates t, r, θ, ϕ, we obtain for the action

$$\begin{aligned}(da)^2 = &\left(\left(m_0 c^2\right)^2 - 2m_0 c^2 \frac{m_0 GM}{r} + \left(m_0 \frac{GM}{r}\right)^2\right)(dt)^2 \\ &- \left((m_0 v)^2 - 2m_0^2 v \frac{GM}{cr} + \left(m_0 \frac{GM}{cr}\right)^2\right)\left\{(dr)^2 + r^2\,(d\Omega)^2\right\}\end{aligned} \tag{4}$$

which, on multiplication by the 'action as a distance' factor $(\kappa_0)^2 = (1/m_0 c)^2$

$$\begin{aligned}(ds)^2 = &\left(1 - 2\frac{GM}{c^2 r} + \left(\frac{GM}{c^2 r}\right)^2\right)(cdt)^2 \\ &- \left(\left(\frac{v}{c}\right)^2 - 2\frac{v}{c}\frac{GM}{c^2 r} + \left(\frac{GM}{c^2 r}\right)^2\right)\left\{(dr)^2 + r^2\,(d\Omega)^2\right\}\end{aligned} \tag{5}$$

contains, in the limit of $r \gg GM/c^2$, the same physical implications as the Schwarzschild metric but with the speed of light always c for all observers.

The negative of the second derivative of a field is proportional to the source, by definition. The source of the "curvature" is either the mass density (in the second term) or the energy of the gravitational field density (in the last term). The energy of the gravitational field, from the second derivative of $\left(GM/c^2 r\right)^2$, turns out to be negative as corresponds to an attractive interaction.

The square root of (4) divided by dt is the ordinary Lagrangian for a particle in a gravitational field.

References

[1] Keller, J.: The Theory of the Electron; A Theory of Matter from START. Foundations of Physics Series 117. Dordrecht: Kluwer Academic Publishers (2001).

[2] Keller, J.: Unification of Electrodynamics and Gravity from START, Annales de la Fondation Louis de Broglie.27(S) 359-410(2002); Keller, J.: Advances in Applied Clifford Algebras. 11 (S2), 183-204 (2001). Keller J.: A Theory of the Neutrino from START, Electromagnetic Phenomena 3,1 (9)122-139 (2003). Keller, J. and Weinberger, P.: A Formal definition of Carriers, Advances in Applied Clifford Algebras. 12 (1), 39–62 (2002).

[3] Keller, J.: Geometrical Principles of Physics, in: *International Conference on Numerical Analysis and Applied Mathematics 2005 (ESCMSE)*, T. E, Simos, editor, (WILEY-VCH, 126-127, 2005).

[4] Keller, J.: Inventio Principia Geometrica Physicae, in the Proceedings of the 7th International Conference on Clifford Algebras and their Applications -ICCA7-, Toulouse, France, May 19-29, 2005. Pierre Anglés , Editor. Birkhauser, Basel